

# Simplified partitioning model to simulate high pressure under-expanded jet flows impinging vertical obstacles

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## Abstract

Various reduced-order models have been developed to quickly model high pressure under-expanded jets. One example is the two-layer partitioning model which was developed to model underexpanded jets, but it has not been evaluated for high pressure jets with obstacles in the jet flow region. This research describes an improved two-layer partitioning model based on the Abel-Noble equation of state that is applied here to model horizontal jet flows impacting a vertical obstacle with validations against high pressure gas experiments, full CFD simulations and a revised notional nozzle model based on the Abel-Noble equation of state. The improved two-layer partitioning model accurately predicts the gas concentrations on the obstacle for a 15 MPa underexpanded jet while consuming much less computational resources and time compared with the full CFD simulation.

*Keywords:* underexpanded jet, two-layer partitioning model, Abel-Noble gas, hydrogen safety

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## 1. Introduction

Hydrogen, with its low density and low volumetric energy density has to be stored at extremely high pressures for commercial use. A typical commercial hydrogen-fueled vehicle stores hydrogen at 70 MPa with the pressures in fueling station tanks typically reaching 35 MPa. Thus, risk assessments of the high pressure hydrogen storage tanks are necessary for safety evaluations. Protective walls are built around hydrogen storage tanks to prevent high pressure jets from leaks from extending far out into the surrounding area and extending the lower flammability region volume. Thus, detailed descriptions of the flow field for a high pressure, underexpanded jet impacting a vertical obstacle are needed to evaluate and design engineered safety solutions.

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11 High pressure underexpanded jet flows will lead to complex nearfield shock structures.  
12 Different pressure ratios between the stagnation pressure and the atmospheric pressure lead  
13 to different underexpanded nearfield shock structures with four kinds of underexpanded  
14 jet flows categorized based on their pressure ratio and, hence, their underexpansion level.  
15 Relatively low pressure, underexpanded jets will form a weak shock will the nozzle exit[1, 2].  
16 As the pressure increases, the jet flow becomes moderately underexpanded with diamond  
17 shaped oblique shocks[1–5]. The oblique shocks, also called the slip region or the barrel  
18 shock region, are reflected to form the reflected oblique shock structure. The reflected  
19 oblique shock will regenerate new intersecting shocks when they reach the outer boundary  
20 of the jet core region. This cell structure in the slip region tends to repeat itself along the  
21 axial direction downstream until the jet reaches ambient pressure. Highly-underexpanded jet  
22 flows are characterized by the appearance of a Mach disk at the end of the supersonic jet core  
23 region inside the oblique shock structure slip region[6–8]. The Mach disk and the reflected  
24 shock intersect at a triple point and the Mach disk is not a perfect flat normal shock but has  
25 some curvature. The flow in the slip region continues at a much higher velocity than the flow  
26 in the core region after the Mach disk. The slip region can continue to be supersonic with  
27 more reflected shocks after the Mach disk location as the slip region expands into the slower  
28 core region until become one flow region. In very high pressure underexpanded jets, the slip  
29 region normally has a larger area than the potential core region[9, 10]. Air is entrained into  
30 the slip region starting shortly after the flow leaves the nozzle with no air entrained into the  
31 core flow region. The total jet diameter decreases in very high pressure, underexpanded jets  
32 as ambient air is entrained into the flow[11].

33 Traditional CFD numerical simulations try to solve the Navier-Stokes equations plus  
34 other conservation equations throughout the whole simulation geometry, which is called the  
35 full CFD simulation model method in this paper. Full CFD simulations are intrinsically  
36 unstable when used to simulate supersonic flow shock structures. They require very long  
37 computational times and large computational resources for complete high pressure gas flow  
38 simulations[12]. Therefore, full CFD simulations are not practical for simulating the large  
39 flow fields of underexpanded jets.

40 In hydrogen safety risk analyses, however, the main interest is the gas concentration pro-  
41 files in the far field of the underexpanded jet. The underexpanded jet flow profile can then be  
42 treated as a relatively simple subsonic compressible flow starting with an already expanded  
43 jet based on assumptions about the conservation of mass, momentum and energy[11]. That  
44 has given rise to the development of various reduced-order notional nozzle models. The key  
45 idea of the notional nozzle models is to predict simplified inlet boundary conditions for un-  
46 derexpanded jet flow simulations. Typical notional nozzle models replace the original high  
47 pressure boundary gas inlet condition with fully expanded low pressure velocity/mass inlet(s)  
48 boundary conditions which eliminates the nearfield shock structure region and avoids the  
49 complex numerical simulations of the shocks. The application of the notional nozzle models  
50 still require CFD simulations throughout the extended flow field model geometry. However,  
51 the simulations are much more efficient without the need to simulate the shock structures  
52 of the high pressure gas inlets. The objective of the simplified notional nozzles is then to  
53 predict reasonable gas flow profiles in the jet flow farfield with much faster calculations.

54 There have been many studies aimed at developing notional nozzle models that give more  
55 accurate predictions with more universally applicable simulation conditions. Thring[13]  
56 developed the first notional nozzle concept by assuming that the notional nozzle had the  
57 same momentum flux and velocity as the real gas nozzle with a gas density equal to that of  
58 the gas at ambient conditions. Their model and all other models also assumed an isentropic  
59 expansion between the stagnation conditions in the tank to the nozzle. Birch et al.[14]  
60 developed a notional nozzle model (Birch84 model) assuming mass conservation between the  
61 real nozzle and the notional nozzle without air entrainment. The notional nozzle pressure and  
62 temperature were assumed to be the ambient conditions and the gas velocity was assumed  
63 to be the local sonic velocity. Many other studies have been inspired by the Birch84 model.  
64 The basic assumptions of the Birch84 model, atmospheric pressure in the notional nozzle and  
65 the jet flow mass conservation with zero air entrainment, were used as the basic assumptions  
66 by many of the following models. Ewan et al.[10] slightly modified the Birch84 model by  
67 assuming that the notional nozzle temperature was equal to the real nozzle gas temperature  
68 with the other assumptions. Gore et al.[15] developed a modified notional nozzle model  
69 based on the momentum conservation assumption between the real nozzle and the notional  
70 nozzle. The notional nozzle diameter was assumed to be the same as the real nozzle but  
71 with the notional nozzle ambient pressure assumption. Birch et al.[16] gave an improved  
72 notional nozzle model (Birch87 model) by adding the momentum conservation assumption  
73 in addition to the original mass conservation assumption. The notional nozzle temperature  
74 was assumed to be equal to the stagnation temperature of the ideal gas. Several studies have  
75 shown that the Birch87 model gives more accurate predictions[16–18]. Yüceil et al.[19] used  
76 the atmospheric pressure assumption and the mass, momentum and energy conservation  
77 equations to derive a notional nozzle model. Harstad et al.[20] used real underexpanded jet  
78 shock structure observations and assumed that the notional nozzle was right after the Mach  
79 disk and shared the same diameter as the Mach disk. The flow between the real nozzle  
80 and the notional nozzle was assumed to be isentropic with the normal shock wave property  
81 relations between the flow before and after the Mach disk used to calculate the notional  
82 nozzle profiles. These notional nozzle models were all derived using the ideal gas equation  
83 of state (EOS), with later models using a real EOS like the Abel-Noble EOS[21–23].

84 These notional nozzle models give a simplified method to derive boundary conditions for  
85 the notional nozzle that are used as inputs to significantly simplified underexpanded jet flow  
86 numerical simulations. However, all these previous notional nozzle models assumed that  
87 the gas had uniform velocity and composition distributions at the notional nozzle which  
88 is not true in high pressure underexpanded jet flows. Observations have shown that real  
89 underexpanded jet flows have complex shock structures, Mach disks and flow stratification  
90 between the slip region and the core region with only some of the flow through the Mach  
91 disk and most of the high pressure jet flow flowing through the surrounding slip region with  
92 air entrainment[24, 25].

93 Li et al.[17] developed a two-layer partitioning model that takes into account the gas  
94 partitioning between the core region and the slip region in real underexpanded jets. The two-  
95 layer partitioning model assumed that the gas flows into two separate regions upon exiting  
96 the orifice with part of the flow accelerating to very high Mach numbers before passing

97 through the Mach disk where the flow becomes subsonic and part of the flow expanding into  
98 the surrounding slip region with air entrainment into this slip region. The gas was modeled  
99 using the Abel-Noble EOS inside the storage tank and the ideal gas EOS after leaving the  
100 real high pressure nozzle. Such models have been validated for high pressure free jets with  
101 stagnation pressures up to 35 MPa. However, given the high stagnation pressure in the tank,  
102 the pressure is still much higher than the ambient pressure when choked at the nozzle, so  
103 the gas inside the Mach region and possibly inside the slip region should not be treated as  
104 an ideal gas. The original two-layer model also did not take into consideration the enthalpy  
105 equation for the Abel-Noble EOS. In addition, the validity of two-layer model for predicting  
106 high pressure jet flows from an orifice that involves any flow geometry other than a free jet  
107 has not been tested.

108 This research presents an improved version of the two-layer partitioning model to more  
109 accurately model high pressure underexpanded jets. The model is applied to the flow of  
110 a horizontal jet impacting a vertical obstacle in the flow field. The simulation results are  
111 compared with both experimental data and simulation results from other models to validate  
112 the partitioning model. The improved two-layer partitioning model can then be used in high  
113 pressure hydrogen simulations to evaluate storage safety.

## 114 2. Model setup

115 Two simplified models were used for the simplified numerical simulations with an im-  
116 proved version of the two-layer partitioning model and a revised version of the traditional  
117 Birch87 notional nozzle model[16]. The Birch87 model was revised by using the Abel-Noble  
118 gas EOS instead of the ideal gas EOS for the density. In both models, the stagnation gas  
119 temperature was assumed to be equivalent to the ambient temperature.

### 120 2.1. Improved two-layer partitioning model

121 The notation for the improved two-layer model is shown in Figure 1[17]. The improved  
122 two-layer model assumes that the pressure ratio is much larger than the critical pressure for  
123 choked flow so the jet flow is extremely underexpanded. In that scenario, the first shock cell  
124 is the dominant shock cell in the nearfield shock structure. Some studies have shown that  
125 the Mach disk should not be assumed to be a normal shock but the curvature should be  
126 taken into account[11]. However, the current study still treats the Mach disk as a normal  
127 shock for simplicity.

128 In the improved two-layer model, the gas was assumed to flow isentropically from stagna-  
129 tion in the tank (state 0) to the real nozzle (state 1). The stagnation pressure was assumed  
130 to be higher than the critical pressure so the gas was choked at the real nozzle. After the real  
131 nozzle, part of the gas flowed through the core region and reached the Mach disk (state 2a)  
132 with the highest velocity occurring just before the Mach disk. After flowing across the shock  
133 wave at the Mach disk, the gas pressure returned to the ambient pressure after the Mach  
134 disk (state 2b). The rest of the gas expanded out into the slip region, also referred to as the  
135 barrel shock region, surrounding the core region with a significant amount of air entrained  
136 into the this slip or mixing region. This gas mixture then expanded to atmospheric pressure

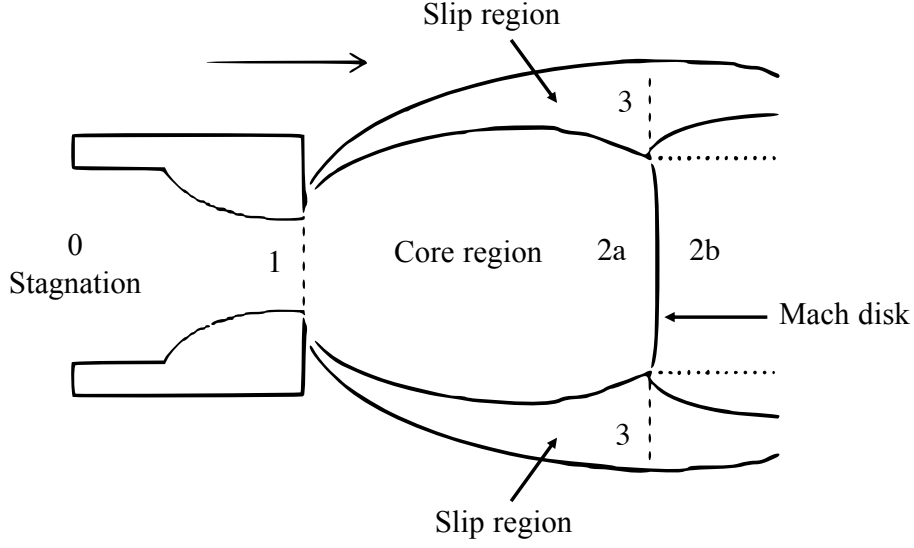


Figure 1: Improved two-layer model notation

137 (state 3). States 2b and 3 formed the two concentric circle notional nozzle regions of the  
 138 two-layer partitioning model. In both sections the pressure was assumed to be atmospheric.  
 139 The flowing gas was modeled using the Abel-Noble EOS throughout the model, while the  
 140 air entrained into the slip region was assumed to be stagnant before being entrained and  
 141 was modeled as an ideal gas.

#### 142 2.1.1. Mass flow rate at the real nozzle

143 The high pressure gas was assumed to flow from state 0 in the tank and was choked at  
 144 the orifice at state 1 with the properties at both states satisfying the Abel-Noble EOS[26]:

$$p_1(v_1 - b) = R_g T_1 \quad (1)$$

145 where  $v$  is the specific volume of the gas which is related to the density as  $v = \frac{1}{\rho}$ , where  $\rho$   
 146 is the gas density and  $b$  is the co-volume coefficient of the gas, which is a constant for each  
 147 Abel-Noble gas.

148 The gas velocity at state 1 is equal to the local sound speed so the Mach number is equal  
 149 to 1. The velocity at state 1 is then given by the sound speed for an Abel-Noble gas[26]:

$$U_1 = \frac{v_1}{v_1 - b} \sqrt{\gamma R_g T_1} \quad (2)$$

150 The gas flow from the tank to the nozzle is assumed to be an isentropic expansion which for  
 151 an Abel-Noble is given by[26]:

$$p_0(v_0 - b)^\gamma = p_1(v_1 - b)^\gamma \quad (3)$$

152 Unlike for an ideal gas, the enthalpy is not just a function of the gas temperature as in an  
 153 Abel-Noble gas. The equation exists[26]:

$$\frac{\partial h}{\partial p} = b$$

154 where  $h$  is the gas enthalpy. The enthalpy equation of the Abel-Noble gas, therefore, is:

$$h = c_p T + bp$$

155 The energy equation for an Abel-Noble gas from state 0 to state 1 is then:

$$c_p T_0 + bp_0 = c_p T_1 + bp_1 + \frac{U_1^2}{2} \quad (4)$$

156 These 4 equations were solved to get the 4 unknowns at the choked nozzle,  $p_1$ ,  $T_1$ ,  $v_1$  and  
 157  $U_1$ . The mass flow rate for the given stagnation pressure and nozzle diameter is then given  
 158 by:

$$\dot{m}_1 = \frac{U_1 \pi d_1^2}{4v_1} \quad (5)$$

### 159 2.1.2. Mach disk boundary conditions

160 After the choked orifice, the high pressure flow was assumed to split into the center Mach  
 161 region and the surrounding slip region. Air was assumed to be entrained into the slip region  
 162 but not the core with no viscous effects between the slip region and the surrounding air.

163 The expansion from state 1 to state 2a was also assumed to be isentropic with conserva-  
 164 tion of the total enthalpy per unit mass from state 1 to state 2a.

$$h_1 = h_2 = c_p T_{2a} + bp_{2a} + \frac{U_{2a}^2}{2} \quad (6)$$

$$p_1(v_1 - b)^\gamma = p_{2a}(v_{2a} - b)^\gamma \quad (7)$$

166 The gas was then assumed to cross the Mach disk before reaching state 2b. The post-  
 167 Mach disk pressure was assumed to be atmospheric. The gas mass, momentum and energy  
 168 are conserved across the Mach disk.

$$\frac{U_{2a}}{v_{2a}} = \frac{U_{2b}}{v_{2b}} \quad (8)$$

$$p_{2a} + \frac{U_{2a}^2}{v_{2a}} = p_{2b} + \frac{U_{2b}^2}{v_{2b}} \quad (9)$$

$$c_p T_{2a} + bp_{2a} + \frac{U_{2a}^2}{2} = c_p T_{2b} + bp_{2b} + \frac{U_{2b}^2}{2} \quad (10)$$

169  $U_{2a}$  was supersonic and  $U_{2b}$  was subsonic.

170 The gas was treated as an Abel-Noble gas at states 2a and 2b.

$$p_{2a}(v_{2a} - b) = R_g T_{2a} \quad (11)$$

$$p_{2b}(v_{2b} - b) = R_g T_{2b} \quad (12)$$

171 The gas properties at states 2a and 2b were found by solving Eqs. 6 to 12. The mass  
172 flow rate through the Mask disk was then calculated using:

$$\dot{m}_2 = \frac{U_{2b} \pi d_m^2}{4v_{2b}} \quad (13)$$

173 The Mach disk diameter,  $d_m$ , was calculated using the equation given by Velikorodny et  
174 al.[27]:

$$\frac{d_m}{d_1} = \alpha \frac{z_m}{d_1} \sqrt{1 - \frac{\gamma + 1}{\gamma} \times \left(\frac{\gamma + 1}{\gamma - 1}\right)^{-0.5}} \quad (14)$$

175 where  $\alpha$  is an empirical constant whose value was set to 0.954 in this research as rec-  
176 ommended by the measurements of the hydrogen Mach disk diameter by Li[28].  $z_m$  is the  
177 location of the Mach disk which was given by the empirical equation from Li et al.[17]:

$$\frac{z_m}{d_e} = 0.67 \sqrt{\frac{p_0}{p_\infty}} \quad (15)$$

178 where  $p_\infty$  is the ambient pressure.

179  $\dot{m}_2$ ,  $T_{2b}$ ,  $z_m$  and  $d_{2a}$  were then used as the Mach disk boundary conditions for the improved  
180 two-layer partitioning model boundary conditions.

### 181 2.1.3. Slip region boundary conditions

182 In the slip region, the high pressure gas was assumed to entrain air that was originally  
183 at zero velocity, atmospheric temperature and atmospheric pressure with the properties  
184 satisfying the ideal gas equation. The gas and the air at state 3 were assumed to be uniformly  
185 mixed with the gas and the air having the same velocity at atmospheric pressure.

186 The slip region thickness was calculated using the empirical equation[29]:

$$\frac{B_m}{z_m} = 0.135 \left[ 1 + \frac{1}{\left(\frac{p_0}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} \left(1 + \frac{\gamma-1}{2}\right)} \right] \quad (16)$$

187 When the gas to be simulated was hydrogen instead of helium, the slip region thickness  
188 was derived from the empirical equation by Li et al.[17]:

$$\frac{B_m}{d_1} = 0.30 \sqrt{\frac{p_0}{p_\infty}} \quad (17)$$

189 The slip region area was then:

$$A_{Bm} = \left(\frac{d_m}{2} + B_m\right)^2 \pi - \frac{d_m^2 \pi}{4} \quad (18)$$

190 The mass, momentum and energy equations were then used to determine the conditions  
 191 of the gas mixture in the slip region:

$$\dot{m}_1 - \dot{m}_2 = \frac{U_3 A_{Bm}}{v_{3,gas}} \quad (19)$$

$$(p_1 - p_\infty) \frac{\pi d_1^2}{4} + \dot{m}_1 U_1 = \dot{m}_2 U_{2b} + U_3^2 A_{Bm} \left( \frac{1}{v_{3,gas}} + \frac{1}{v_{3,air}} \right) \quad (20)$$

$$(\dot{m}_1 - \dot{m}_2) \left( c_p T_1 + b p_1 + \frac{U_1^2}{2} \right) = U_3 A_{Bm} \frac{c_p T_3 + b p_{3,gas} + \frac{U_3^2}{2}}{v_{3,gas}} + \frac{c_{p3,air} (T_3 - T_\infty) + \frac{U_3^2}{2}}{v_{3,air}} \quad (21)$$

192 The high pressure gas properties were based on the Abel-Noble EOS, while the entrained  
 193 air properties were based on the ideal gas EOS:

$$p_{3,gas} (v_{3,gas} - b) = R_g T_3 \quad (22)$$

194

$$p_{3,air} v_{3,air} = R_{g,air} T_3 \quad (23)$$

195 where  $v_{3,gas}$  and  $v_{3,air}$  are the specific volumes of the gas and air and  $p_{3,gas}$  and  $p_{3,air}$  are the  
 196 partial pressures of the gas.

197 The mixture of helium and air satisfies:

$$p_{3,gas} + p_{3,air} = p_3 \quad (24)$$

198 The slip region gas conditions were then calculated by solving Eqs. 19 to 24. The mass  
 199 flow rate in the slip region,  $\dot{m}_3$ , and the gas mass fraction,  $Y$ , were then calculated using:

$$\dot{m}_3 = U_3 A_{Bm} \left( \frac{1}{v_{3,gas}} + \frac{1}{v_{3,air}} \right) \quad (25)$$

200 and:

$$Y = \frac{U_3 A_{Bm}}{\dot{m}_3 v_{3,gas}} \quad (26)$$

201  $\dot{m}_3$ ,  $T_3$ ,  $B_m$ ,  $z_m$  and  $Y$  were then used as the inlet conditions for the slip region inlet  
 202 boundary conditions of the improved two-layer partitioning model.

203 The stagnation pressure and temperature in the tank, the nozzle diameter and the am-  
 204 bient pressure were then used to calculate all the parameters for both the Mach disk region  
 205 and the slip region for the simulations.



206 *2.2. Revised Birch87 model*

207 The traditional notional nozzle model developed by Birch et al.[16] was extended to  
 208 simulate the high pressure underexpanded jet flow. In the original Birch87 model, the gas  
 209 was assumed to be an ideal gas. In the current work, the Abel-Noble real gas EOS was used  
 210 to take into account the real gas effects due to high pressure.

211 In the revised Birch87 model, the gas was assumed to flow isentropically from the stag-  
 212 nation tank (state 0) to the real nozzle where the flow was choked (state 1). The gas flow  
 213 was then assumed to have a uniform velocity at the notional nozzle downstream of the real  
 214 nozzle. The mass and momentum were both conserved between the real nozzle and the  
 215 notional nozzle. The temperature and pressure of the notional nozzle were assumed to be  
 216 the known ambient conditions. With the Abel-Noble EOS, the conditions at the notional  
 217 nozzle (state n) satisfy:

$$p_n(v_n - b) = R_g T_n \quad (27)$$

218 where  $p_n = p_\infty$  and  $T_n = T_\infty$ .

219 The mass conservation equations are:

$$\dot{m}_1 = \dot{m}_n = \frac{U_n \pi d_n^2}{4v_n} \quad (28)$$

220 while the momentum conservation equation is:

$$(p_1 - p_\infty) \frac{\pi d_1^2}{4} + \dot{m}_1 U_1 = \dot{m}_n U_n \quad (29)$$

221 The notional nozzle diameter,  $d_n$ , and the velocity,  $U_n$ , were then determined from Eqs.  
 222 27 to 29, with these conditions used for the notional nozzle conditions in the Abel-Noble  
 223 Birch87 model. The Birch87 model neglects the axial distance between the real nozzle and  
 224 the notional nozzle.

225 **3. Experimental design**

226 The experimental system shown in Fig. 2 produced high pressure, underexpanded jet  
 227 flows at pressures up to 50 MPa with measurements of the gas concentrations along the  
 228 obstacle plate. In the experiments, helium was used in lieu of hydrogen due to its stable  
 229 chemical properties and because its physical properties are similar to those of hydrogen.  
 230 Helium was pumped from storage tanks by a nitrogen powered gas booster into a carbon-  
 231 fiber tank with a design pressure of 70 MPa. An ER5000 electronic valve then controlled the  
 232 inlet pressure into the very short nozzle. The 30 cm × 30 cm × 0.6 cm obstacle plate was placed  
 233 vertically in the flow field at the desired distance from the nozzle exit. The plate center was  
 234 placed at the jet flow centerline with a laser alignment system used to align the plate in the  
 235 horizontal and vertical directions. A level was used to ensure that the plate was vertical. Six  
 236 XEN-TCG3880Pt thermal conductivity sensors with accuracies of 2% were mounted on the  
 237 plate along the vertical direction. The sensors were placed along the vertical centerline 6 cm  
 238 and 9 cm above and 3 cm, 6 cm and 9 cm below the plate center. The sensors were recessed

239 halfway through the back side of the plate with a 1 mm diameter hole then drilled the rest of  
 240 the way through the plate. The small diameter was used to reduce the effects of convection  
 241 in the sensor. All the sensors were connected to an Agilent 34970A data acquisition system  
 242 with their calibrated voltage signals then converted to helium concentrations. The sensors  
 243 were calibrated using four standard helium-air concentrations. A 0.5 mm diameter nozzle  
 244 was used for the experiments. The concentrations were measured for nozzle-plate distances  
 245 of 0.2 m, 0.3 m and 0.4 m and for a stagnation pressure of 15 MPa.

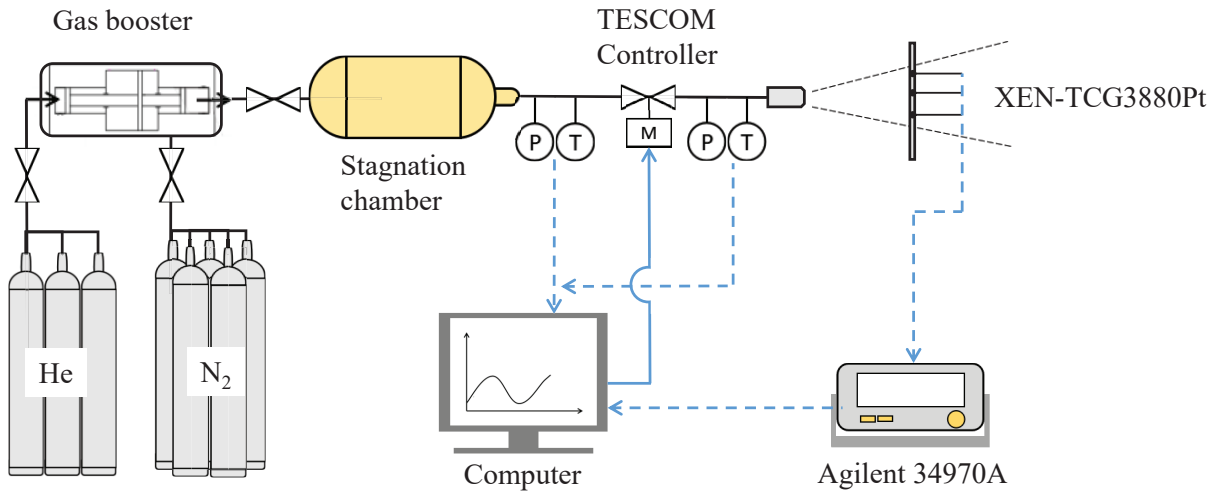


Figure 2: Experimental system layout

#### 246 4. Numerical Simulations

247 The flow fields for the high pressure underexpanded jet flows around the plate were  
 248 modeled with both the full CFD simulation models and the two simplified reduced-order  
 249 models to predict the helium distributions around the vertical plate. The improved two-  
 250 layer model was then used to predict the hydrogen profiles around the vertical plate. All  
 251 the models solved the 3-D Navier-Stokes equations with the  $k-\omega$  turbulence model and the  
 252 energy and species equations. Three dimensional numerical simulation geometries was set  
 253 up for all three simulation models, with the  $x$  direction representing the axial direction,  $y$   
 254 representing the vertical direction and  $z$  representing the radial direction. The  $z = 0$  plane  
 255 was used as symmetry to reduce the computational cost with only half of the region ( $z > 0$   
 256 region) modeled in each case. The simulation started from the inlets of each simulation  
 257 case and extended 3 cm beyond the outside edges of the plate in all three directions to  
 258 more accurately predict the gas concentrations along the plate. Fluent 16.0 was used as the  
 259 numerical solution software. Structured hexagonal meshes were used in all the simulations  
 260 with the second-order upwind discretization scheme and the  $k-\omega$  turbulence model. Mesh  
 261 independence checks were done for each case with the results given for the optimal number  
 262 of elements in each case. All of the external boundaries in all of the models were set as

263 outlet boundary conditions with zero gauge pressures. The gravity effect was included in  
264 all the simulations. The simulations used a compressible air-gas mixture with the density,  
265 specific heat, thermal conductivity and viscosity all modeled by ideal gas mixing laws.

#### 266 4.1. Full CFD Simulations

267 A diagram of the full CFD simulation geometry and mesh is shown in Figure 3 with an  
268 expanded view of the mesh near the inlet.

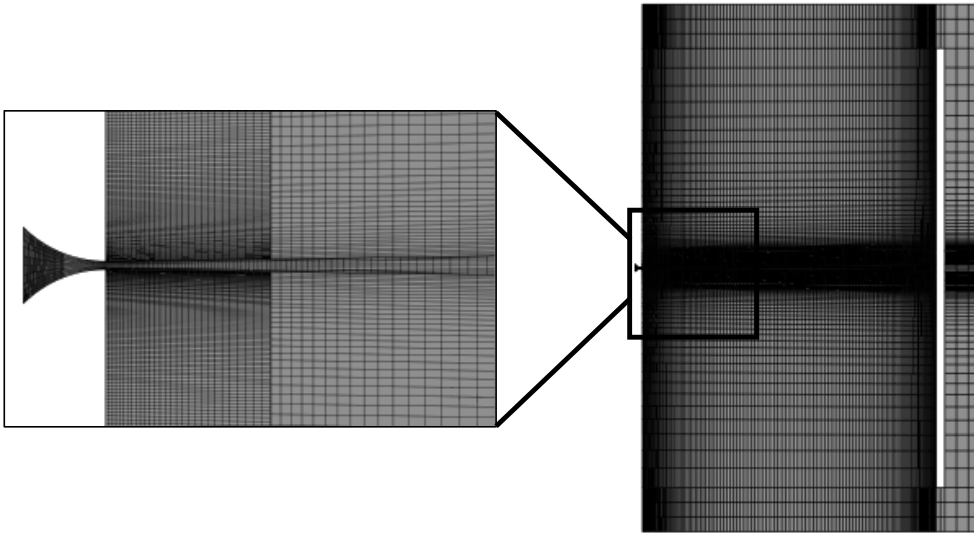


Figure 3: Full CFD model mesh diagram

269 Since the full CFD simulation model required that the geometry include the whole flow  
270 region, the simulation geometry started from the stagnation high pressure inlet. The inlet in  
271 the full CFD model simulation geometry was designed as a converging nozzle to improve the  
272 flow field at the nozzle exit with the mesh strongly refined in the near nozzle region and near  
273 the plate region to improve the accuracy and the convergence. The inlet boundary condition  
274 was set to be a pressure inlet with a gauge pressure of 15 MPa and a temperature of 300 K  
275 with pure helium or hydrogen. The nozzle exit was set to be located at the  $x = 0$  plate  
276 with the nozzle center at the origin. The nozzle diameter was set to be 0.5 mm. The density  
277 solver was used with the Courant number initially set to 0.1 due to the extremely unstable  
278 calculation and slowly increased up to 5 as long as the calculation remained stable. The  
279 flow was choked at the nozzle exit and then expanded to a maximum Mach number of about  
280 11 just before the Mach disk. Thus, the flow field included the supersonic jet, the Mach  
281 disk and the barrel shocks in the slip region around the supersonic jet. The meshes differed  
282 for each nozzle to plate distance but all had 1 to 3 million elements. Mesh independence  
283 studies for each plate distance showed that meshes with 1.5 to 2 million elements gave mesh  
284 independent results. The  $y_+$  along the plate surface were all less than 5.

285 *4.2. Simplified Model Simulations*

286 The flow field was also modeled numerically using CFD models with the flow input  
 287 boundary conditions specified by the two simplified models described in Section 2. The  
 288 simulation geometries for both simplified models did not include the high pressure supersonic  
 289 region but started with the notional nozzle boundary inlets. Therefore, the whole simulation  
 290 geometry was cubic and slightly smaller than the full CFD simulation geometry in each case.  
 291 The mesh details near the inlets for the two simplified models are shown in Figure 4.

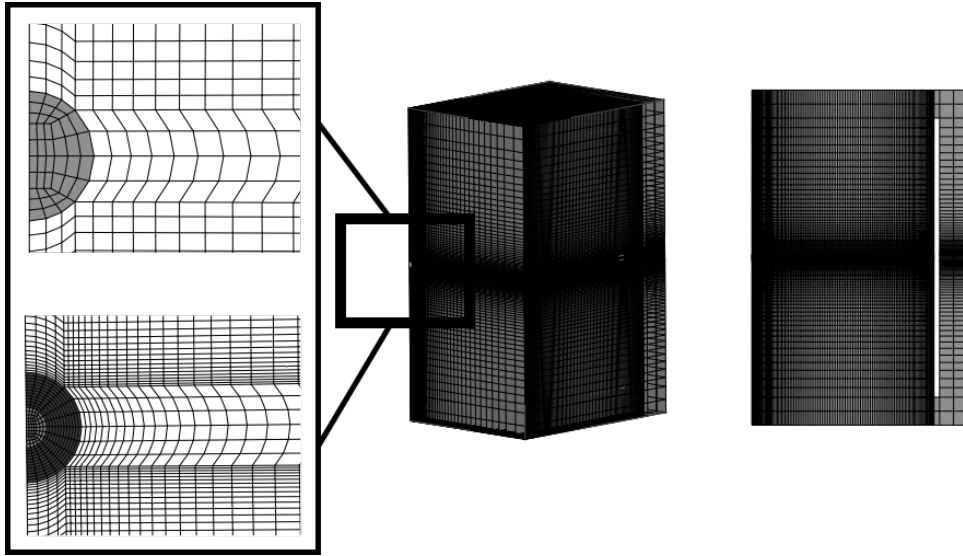


Figure 4: Simplified models mesh diagram: revised Birch87 model (upper-left); improved two-layer model (lower-left); symmetric view (right)

292 For the improved two-layer model, the boundary inlet condition included two concentric  
 293 inlets for the Mach disk inlet (light half-circle inlet inside) and the slip region inlet (dark  
 294 annulus inlet outside the Mach disk inlet). Both inlets had the same axial distance from  
 295 the real nozzle exit. For the revised Birch87 model, the boundary inlet conditions included  
 296 one notional nozzle mass inlet. The sole notional nozzle inlet was located at the  $x = 0$   
 297 plane with the center at the origin, as in the revised Birch87 notional nozzle the axial  
 298 distance between the real nozzle and notional nozzle neglected as normal. The pressure  
 299 solver was used for both simplified model cases with the density solver used in some cases as  
 300 validation. The inlet conditions for the improved two-layer model and the revised Birch87  
 301 model cases were both much slower than the velocities in the core region before the Mach  
 302 disk, so the solutions with the two simplified models were much faster and more stable.  
 303 The revised Birch87 model simulation geometry had approximately 700,000 elements while  
 304 the improved two-layer partitioning model simulation geometry had approximately 600,000  
 305 elements after the mesh independence studies. The  $y_+$  along the plate surface were all less  
 306 than 5 for better simulation results.

307 **5. Results and Discussions**

308 *5.1. Helium simulations and validation*

309 The nozzle sizes and flow rates for boundary conditions of both the revised Birch87 model  
 310 and the improved two-layer model are listed in Table 1 for helium flow through a 0.5 mm  
 311 nozzle from a tank at 15 MPa and 300 K stagnation conditions. The Mach disk in the  
 312 improved two-layer model is smaller than the notional nozzle in the revised Birch87 model  
 313 with only 6.8% of the total helium flow rate flowing through the Mach disk (the total helium  
 314 flow rate through both parts of the improved two-layer model is the same as the helium flow  
 315 rate through the revised Birch87 notional nozzle). The gas flow rate through the slip region  
 316 then includes a significant amount of entrained air. The helium molar fraction in the slip  
 317 region is more than 90%. Given the high molar fraction of helium in the slip region, the  
 318 notional nozzle obtained using the improved two-layer model can be viewed as a traditional  
 319 notional nozzle that includes both non-uniform gas velocities and air entrainment. The total  
 320 diameter of improved two-layer model is less than that given by the revised Birch87 notional  
 321 nozzle model (2.950 mm for the two-layer model compared with 3.811 mm for the Birch87  
 322 model). The axial distance of the notional nozzle from the real nozzle is 4.076 mm for the  
 323 two-layer model and 0 for the revised Birch87 model.

Table 1: Flow parameters for the two reduced-order models for helium at a stagnation pressure of 15 MPa, stagnation temperature of 300 K and an actual nozzle diameter of 0.5 mm

	Improved two-layer partitioning		Revised Birch87
	Mach disk region	slip region	notional nozzle
diameter/thickness (mm)	1.738	0.606	3.811
gas mass flow (kg/s)	$1.81 \times 10^{-4}$	$4.43 \times 10^{-3}$	$2.66 \times 10^{-3}$
gas temperature (K)	288.5	94.3	300.0
gas mass fraction	1.0	0.56	1.0

324 The helium simulation results and the experimental data along the vertical plate center-  
 325 line are compared in Fig. 5 for various obstacle-nozzle distances. The error bars represent  
 326 the mean standard errors of 3 sets of measurements. With the measurement uncertainty  
 327 taken into account, the measured helium mass fractions along the centerline are nearly sym-  
 328 metric with the measured helium mass fractions decreasing as both the nozzle-plate distance  
 329 and the distance from the plate center increase. Throughout the region of interest, the flow  
 330 is within the momentum dominated region and the measured points show little effect of  
 331 buoyancy, even for the 40 cm nozzle-plate spacing.

332 Both the full CFD simulation case and two simplified model cases give reasonable pre-  
 333 dictions of the helium mass fraction trends along the vertical plate centerline as shown in  
 334 Fig. 5. All three simulation results show that the helium mass fraction profiles along the  
 335 center vertical line of the obstacle plate are similar to Gaussian distributions. The increase  
 336 in the axial distance between the nozzle and the obstacle plate not only reduces the on-plate  
 337 helium concentrations, but also smooths the distributions. The concentrations drop faster

338 near the vertical centerline than near the lower and upper edges as the nozzle-plate distance  
 339 increases.

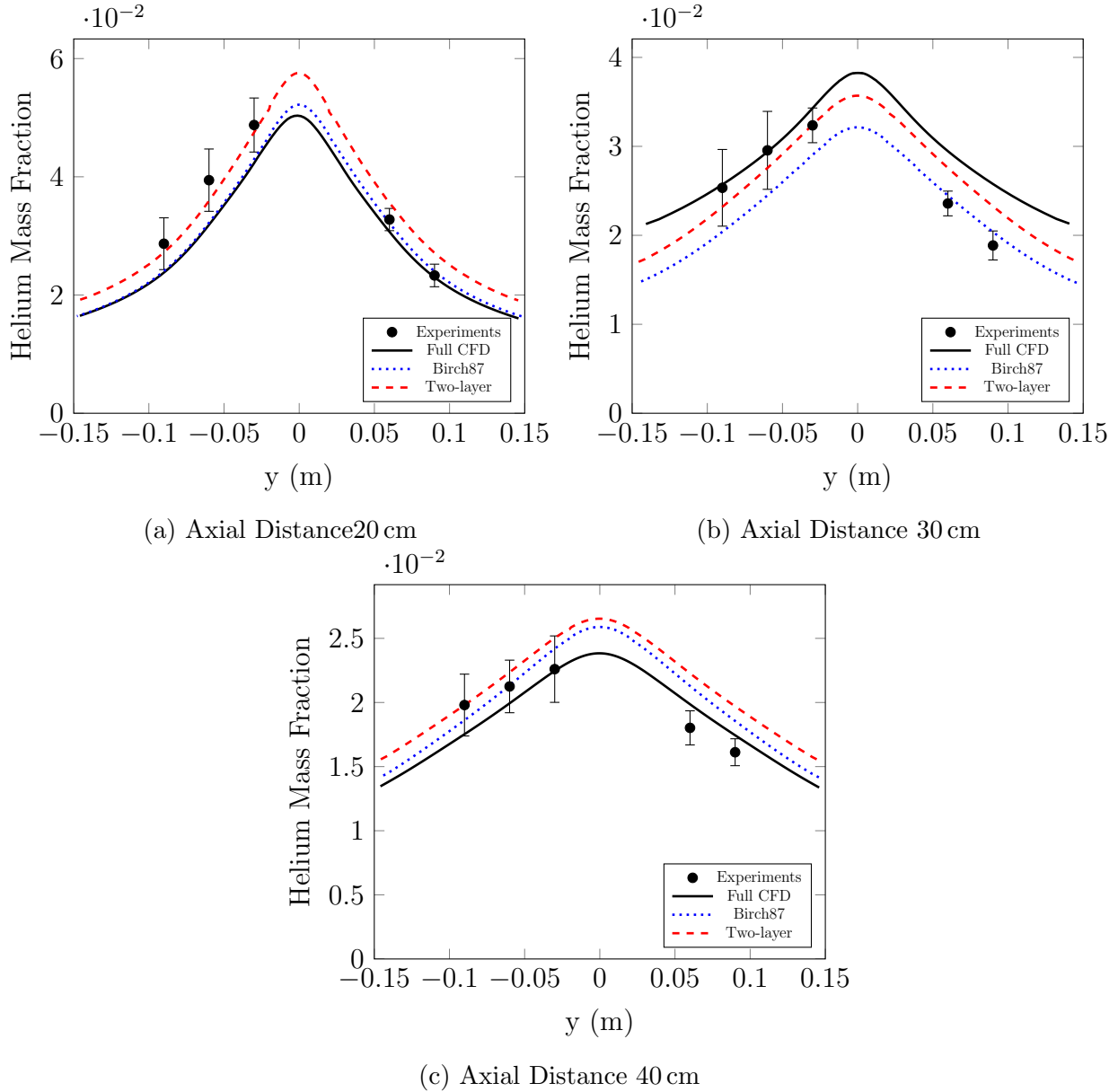


Figure 5: Measured and predicted helium mass fractions along the vertical plate centerline

340 The predictions of the three models are similar. The improved two-layer model case pre-  
 341 dictes higher concentration profiles than the revised Birch87 model case, with the predictions  
 342 of the full CFD simulations being both lower and higher than those of the reduced-order  
 343 models. The improved two-layer model gave better predictions in the lower plate region,  
 344 while the revised Birch87 model gave better simulation results in the upper plate region.  
 345 The improved two-layer model gives the best simulation results for the nozzle-plate distance

346 of 20 cm but the simulation accuracy decreases as the axial distance between the real nozzle  
 347 and the obstacle plate increases. The difference in the accuracies of the three simulation  
 348 results might also be due to systematic errors in the high pressure experimental system due  
 349 to errors in positioning the plate and errors in the helium concentration measurements.

350 For the underexpanded helium jet flow with the obstacle plate, the two-layer simplified  
 351 model gives good predictions of the on-plate helium concentration profiles. Furthermore,  
 352 both of the reduced order models required significantly less computing time, on the order of  
 353 an hour, than the full CFD model (1 ~ 2 weeks) when simulating the geometry with almost  
 354 the same volume flow region. The simplified models had less elements in the mesh and were  
 355 able to use the pressure solver which converges much faster. Thus, these results verify the  
 356 accuracy and efficiency of the improved two-layer model for simulating high pressure helium  
 357 jet flows with obstacles.

## 358 5.2. Hydrogen simulations

359 The nozzle sizes and flow rates for both the revised Birch87 model and the two-layer  
 360 model are listed in Table 2 for hydrogen flow through a 0.5 mm nozzle from a tank at  
 361 15 MPa and 300 K stagnation conditions. The parameters are similar to those for the helium  
 362 flows. The flow in the Mach disk region accounts for only 8.7% of the total hydrogen mass  
 363 flow through the nozzle. The hydrogen mass fraction in the slip region is 52% (94% molar  
 364 fraction). As in the helium results, the total diameter of the improved two-layer model is  
 365 smaller than the diameter of the revised Birch87 model (3.45 mm for the two-layer model  
 366 compared with 3.606 mm for the Birch87 model). The axial distances of both simplified  
 367 models were the same as for the helium gas models because the Mach disk location is only  
 368 a function to the pressure ratio as shown in Eq. 15.

Table 2: Flow parameters for the two reduced-order models for hydrogen at a stagnation pressure of 15 MPa, stagnation temperature of 300 K and an actual nozzle diameter of 0.5 mm

	Improved two-layer partitioning		Revised Birch87
	Mach disk region	slip region	notional nozzle
diameter/thickness (mm)	2.130	0.660	3.606
gas mass flow (kg/s)	$1.52 \times 10^{-4}$	$3.08 \times 10^{-3}$	$1.75 \times 10^{-3}$
gas temperature (K)	298.4	136.1	300.0
gas mass fraction	1.0	0.52	1.0

369 The improved two-layer model predictions for the high pressure hydrogen underexpanded  
 370 jet flow are shown in Fig. 6 for various nozzle-plate distances. As with the helium simulation  
 371 results, the predicted hydrogen mass fractions along the vertical centerline of the plate are  
 372 symmetric around the center. The hydrogen concentration profiles along the plate surface  
 373 are similar to the simulated helium profiles in Fig. 5 with the hydrogen mass fractions  
 374 also having shapes similar to Gaussian distributions. When the nozzle-plate axial distance  
 375 increases the on-plate hydrogen concentration decreases. The region near the center again

376 has larger hydrogen concentration decreases than the changes near the upper and lower  
 377 parts.

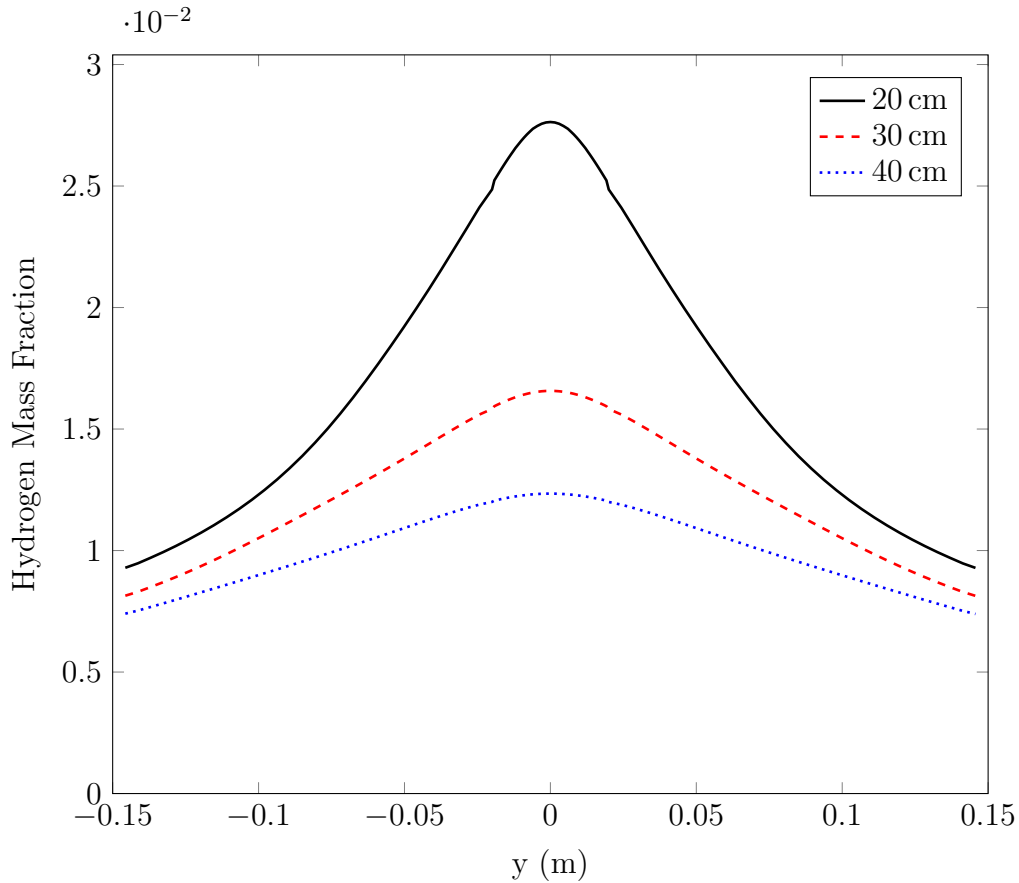


Figure 6: Predicted hydrogen mass fractions along the vertical plate centerline

377  
 378 As shown in Fig. 7, the helium and hydrogen molar concentration profiles along the  
 379 obstacle vertical centerline predicted by the improved two-layer model are similar. The on-  
 380 plate improved two-layer predicted hydrogen molar concentrations are slightly lower than  
 381 the predicted helium concentrations. The differences in the molar concentration increase as  
 382 the nozzle-plate axial distance increases. This phenomenon is caused by the transition from  
 383 momentum-dominated flow to buoyancy-dominant flow. When the nozzle-plate distance is  
 384 small, the jet flow is mainly determined by the initial jet momentum and the gas properties  
 385 have little effect on the concentration profiles in this region. As the nozzle-plate distance  
 386 increases, the flow transitions from momentum-dominated to buoyancy-dominated and the  
 387 gas physical properties have a greater effect on the gas concentration profiles. Given the  
 388 ability of the two-layer model for simulating the helium concentration profiles, the similarities  
 389 between the physical properties of helium and hydrogen and the similarities between the  
 390 predicted profiles, the hydrogen concentrations predicted by the improved two-layer model  
 391 can be regarded as reliable. Thus, the two-layer model can be used for hydrogen safety  
 392 assessments due to its accuracy and low computational cost.



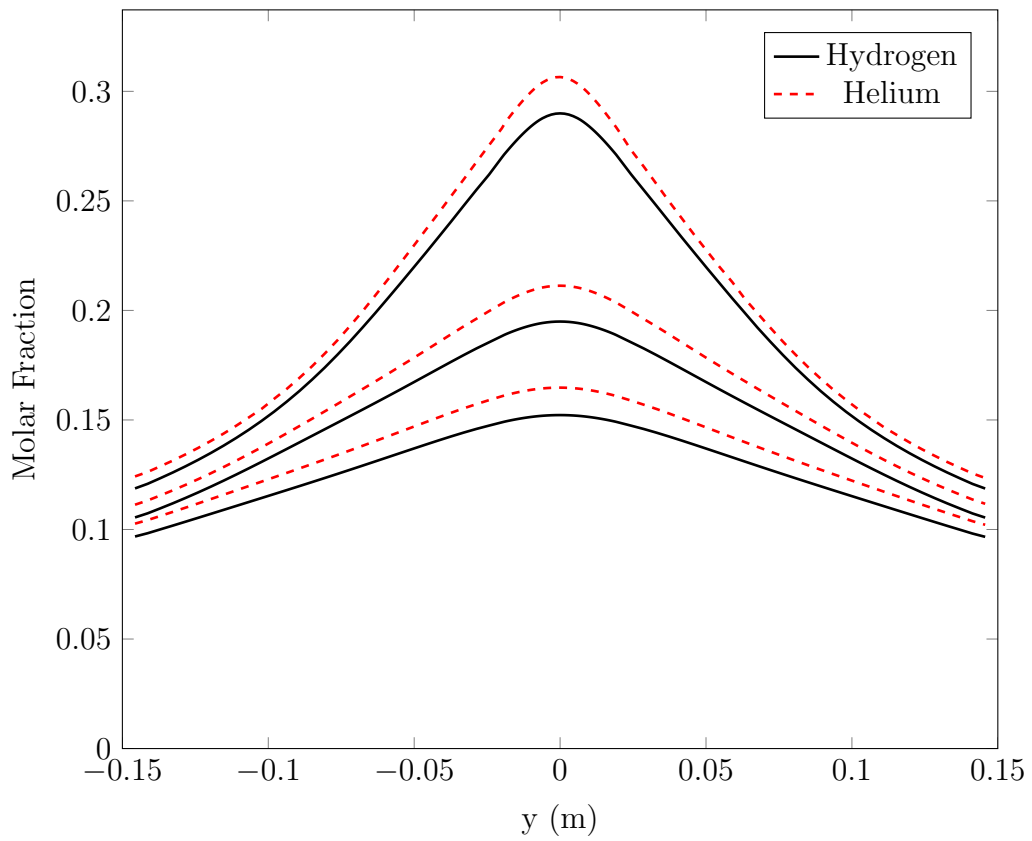


Figure 7: improved two-layer model predicted hydrogen and helium molar concentration along the vertical plate centerline; solid line from top to bottom: hydrogen profiles with axial distance 20, 30 and 40 cm; dashed line from top to bottom: helium profiles with axial distance 20, 30 and 40 cm

## 6. Conclusions

An improved two-layer partitioning model was developed to simplify the CFD calculation and more efficiently simulate high pressure underexpanded jet flows with a vertical obstacle in the flow field. This improved two-layer model was used to simulate high pressure helium and hydrogen jet flows with a vertical plate in the flow field, along with a full CFD model and an revised version of the Birch87 model which uses the Abel-Noble real gas EOS to take into consideration the real gas properties. The helium simulation results were validated against measured helium mass fraction data for a high pressure jet at 15 MPa. All three models predicted the overall trends of the helium concentration profiles on the plate surface. The predictions of the improved two-layer model had good agreement with the experimental data. The validated improved two-layer model was then used to simulate hydrogen high pressure, underexpanded jet flows with a vertical plate in the flow field. The predicted hydrogen molar concentration profiles were similar to the predicted helium profiles. The two-layer model was also significantly faster than the full CFD simulation (hundreds of times faster) and can serve as a cost-effective method to predict hydrogen leak profiles for hydrogen safety and risk assessment studies.

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